

Very preliminary

An Essay on Public Debt Sustainability:
Why Japanese Government Does Not Go Bankrupt?

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Summary

From late 1990s, the government of Japan has been running large fiscal deficit and the public debt piles to an extraordinary extent. Additionally, to prevent infection of COVID-19 and to mitigate its economic impact, the significant fiscal expenditure has been required from 2020. Some economists thus express strong skepticism on public debt sustainability in Japan. This paper explores public debt sustainability from three viewpoints: 1) surveying some methodologies on estimating public debt sustainability using timeseries data, mainly developed by Hamilton and Flavin (1986), including stochastic debt sustainability analysis (SDSA) related to calculate gamble probability developed by Ball et al. (1998); 2) discussing two exceptional cases for public debt sustainability such as the dynamic efficiency/inefficiency, which describes relation between interest rate and growth, and Ricardian equivalence where all of economic agents precisely forecast the future; and, 3) examining debt sustainability in Japan, in particular, the reason why Japanese government does not go bankrupt. The conclusion derived by the paper stresses two points: 1) public debt sustainability in Japan has been supported by difference between interest rate and growth and the primary surplus but mainly by the former; and, 2) public debt sustainability can be analyzed adopting the mainstream economics.

JEL Classifications: C22, E43, H62 and H68

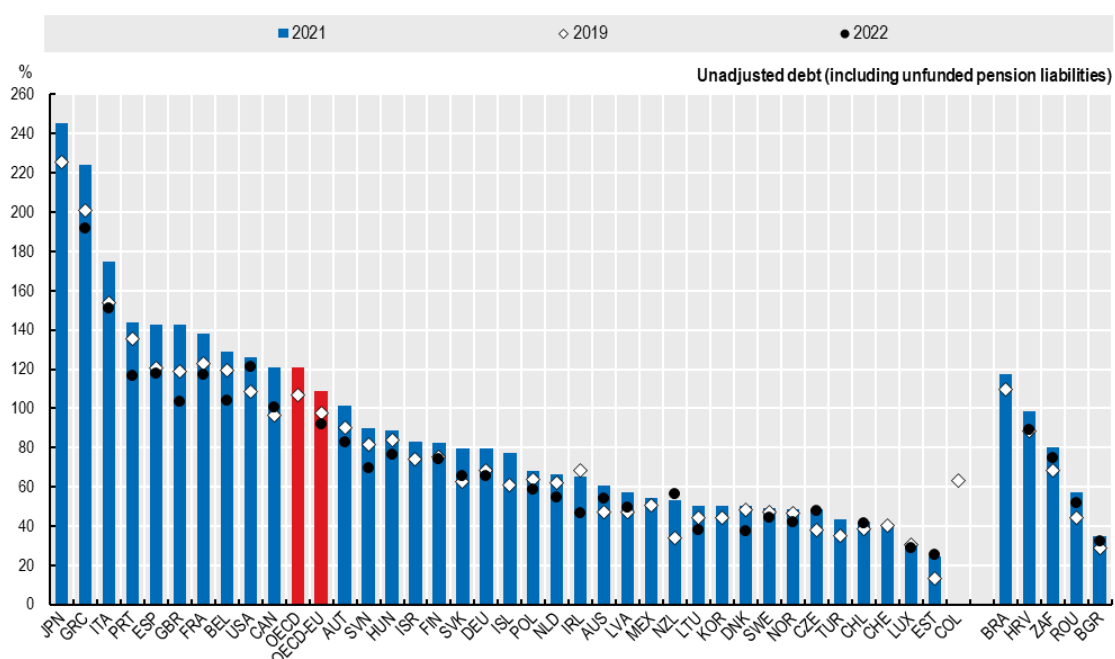
Keywords: Public Debt Sustainability, Timeseries Testing, Ricardian Equivalence, Dynamic Inefficiency, Japan

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1. Introduction

After the bubble burst in early 1990s, the government of Japan has been running huge deficit and its public debt has piled to extraordinary extent. Taking some fiscal measures for COVID-19 aiming at the prevention from its infection and the mitigation for its negative economic impact, the public deficits have reached at enormous level and the public debt is piling up around the world including Japan.¹ Among developed countries, it is widely acknowledged that the government of Japan remarks one of the nations that record the largest debt ratio over GDP as OECD (2023) reports. Figure 1 depicts the general government gross debt as a percentage of GDP.

Figure 1: general government gross debt as a percentage of GDP

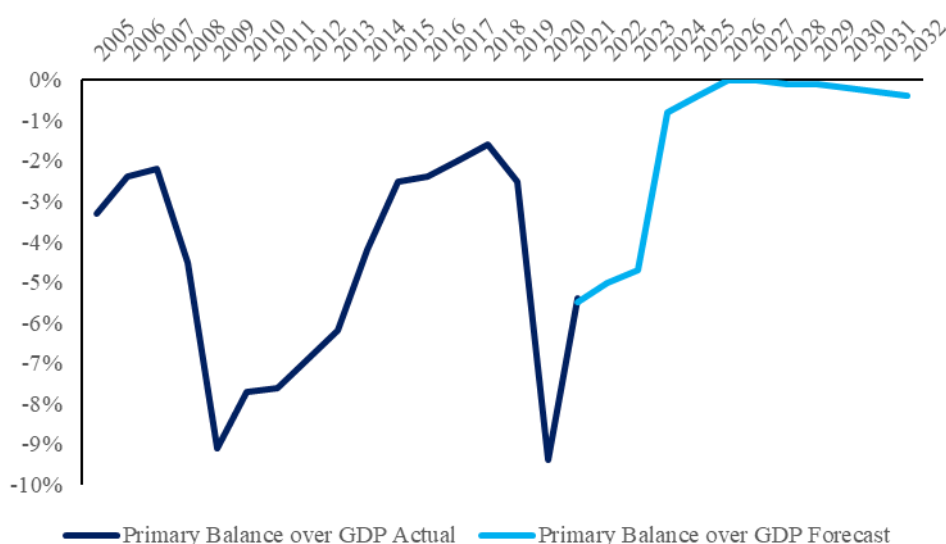


source: OECD (2023), 10.9. General government gross debt as a percentage of GDP, 2019, 2021 and 2022 (p.153)

In general, public debt is widely regarded as bad, as mortgaging the future, or government borrowing would cost our children/grandchildren. Public debt and fiscal deficit must be, however, analyzed from the viewpoint of economic welfare, i.e., from both sides of cost and benefit. Blanchard (2022), e.g., suggests that debt might indeed be good under the assumption of certainty. Contrary in Japan, the cost of public debt/deficit is overstated while its benefit is paid very little attention. Some Japanese media widely stresses that public debt would bring some economic turmoil such as extraordinary inflation, capital flight, enormous depreciation of local currency, and so on. Some economists also undervalue benefits of fiscal deficit that mitigate GDP gap in case of deflation as the functional fiscal policy regime of Lerner (1943) insists.²

Focusing on Japanese fiscal policy, under the former Koizumi cabinet, we faced some confusing discussion on public debt sustainability, especially on relationship between interest rate and growth. At the Council on Economic and Fiscal Policy (CEFP) in February 2006 as Yoshioka (2009) addresses, the debate unfolded between Mr. Takenaka and Professor Yoshikawa. Mr. Takenaka was then the Minister of Internal Affairs and Communications and insisted that interest rates subsequently remained in fact below the growth rate based on actual economic statistics. Mr. Yoshikawa was a private-sector member of the CEFP representing academia in economics, and revealed his opinion based on the dynamic efficient economy where interest rates should exceed growth rate. After this meeting, the “Integrated Reform of Expenditures and Revenues” was discussed as one of important fiscal issues in June 2009 at the CEFP. Later, the outlook for the ratio of the primary balance of the central and local governments to nominal GDP has been estimated as a trial calculation semiannually. Figure 2 shows the Japanese government primary balance over GDP reported by SNA statistics and by CAO (2023).

Figure 2: actual and projection of general government primary balance as a percentage of GDP



source: SNA statistics³ and CAO (2023)

note: 1) Actual statistics until fiscal 2021 is calculated on a general government basis while estimation from fiscal 2022 on is based on sum of central and local governments excluding social security fund.

2) Forecast data are of baseline case.

The paper explores the debate on public debt sustainability, especially from the viewpoint of primary balance and difference between interest rate and growth, and surveys tests using timeseries data. Some exceptional cases such as dynamic inefficiency and Ricardian equivalence will be also referred because these two cases would unconditionally ensure public debt sustainability. Considering the theme of the paper, two criteria have been adopted to economic literature: 1) convergence in zero of

discounted present value of public debt; and 2) stabilization of public debt ratio over GDP. The former is called transversality condition⁴ while the latter is Domar condition, as focused on later. The paper takes the latter condition because entire redemption of government bonds does not seem desirable or realistic. Schinasi et al (2001), e.g., address that the government bonds do function mainly as pricing and quotation of private fixed-income instruments, collateralizing counterparty risks, and so on. In particular, the roles played by US treasury securities may not be easily or fully substituted by private financial instruments since they are issued in US dollar, which is the key currency in international transactions. Apart from this introduction chapter, the paper consists of following four chapters: the second chapter overviews typical discussions on public debt sustainability from the viewpoint of Domar condition; the third chapter surveys the methodologies for public debt sustainability using timeseries data; the fourth chapter deals with exceptional two cases for public debt sustainability, i.e., dynamic inefficiency and Ricardian equivalence; the fifth chapter analyzes public debt sustainability in Japan; and, the last chapter briefly concludes the paper.

2. Overview of Discussion on Public Debt Sustainability

On fiscal sustainability, firstly, Domar (1944) proposes very simple view in definitive equation as follows:

$$(EQ-1) \quad D - D_{-1} = \Delta D = G - T + rD$$

where D public debt or outstanding government bond
 G government expenditure excluding interest payment for bonds
 T government revenue excluding bond issuing
 r interest rate on government bond (decimal)

In (EQ-1), $D - D_{-1} = \Delta D$ is identical to the amount of newly issued outstanding government bond; $G - T$ is to the primary balance;⁵ and rD apparently corresponds to the interest payment on government bond, respectively. Based on (EQ-1), simply differentiating public debt ratio over nominal GDP, following (EQ-2) could be obtained:

$$(EQ-2) \quad \Delta \left(\frac{D}{Y} \right) = \frac{\Delta D}{Y} - \frac{\Delta Y}{Y} \cdot \frac{D}{Y} = \frac{\Delta D}{Y} - g \frac{D}{Y}$$

where G nominal gross domestic product (GDP)⁶
 g nominal GDP growth rate ($\equiv \Delta Y/Y$) (decimal)

Substituting (EQ-1) into (EQ-2), Domar condition is obtained as follow:

$$(EQ-3) \quad \Delta \left(\frac{D}{Y} \right) = \frac{G-T}{Y} + r \frac{D}{Y} - g \frac{D}{Y} = \frac{G-T}{Y} + (r - g) \frac{D}{Y}$$

According to (EQ-3), in other words, to Domar condition,⁷ one of the most important indicators,⁸ which is public debt ratio over GDP depends on two factors, i.e., the primary balance⁹ and difference between interest rate and growth as follows:

- 1) The public debt sustainability improves when the primary balance is positive, i.e., government revenue excluding bond issuing exceeds government expenditure excluding interest payment on bonds and at the same time, interest rate is lower than growth;
- 2) Contrary, the public debt sustainability deteriorates when the primary balance is negative, i.e., government expenditure excluding interest payment on bonds exceeds government revenue excluding bond issuing and at the same time, interest rate is higher than growth;
- 3) The result will be mixed when the primary balance is positive and interest rate is higher than growth and when the primary balance is negative and interest rate is lower than growth.

Summarizing above relation based on the Domar condition, following Table 3 could be obtained.

Table 3: Public Debt Sustainability and Domar Condition

Sign of the first term of righthand Domar condition, i.e., $\frac{G - T}{Y}$	sign of the second term of righthand Domar condition, i.e., $(r - g) \frac{D}{Y}$	public debt sustainability, i.e., $\Delta \left(\frac{D}{Y} \right)$
negative (primary balance is positive)	negative (interest rate is lower than growth rate)	improve
positive (primary balance is negative)	positive (interest rate is higher than growth rate)	deteriorate
positive	negative	mixed
negative	positive	mixed

source: author

Formerly referred debate between Mr. Takenaka and Professor Yoshikawa focused on the second term of the right hand of (EQ-3), i.e., $(r - g)$. In definition, the sign of this term is identical to the issue of dynamic efficiency/inefficiency. When the interest rate exceeds the growth, the economy is regarded as efficient in the sense of Pareto, satisfying dynamic efficiency. Conversely, when the growth rate exceeds interest rate, the economy goes dynamic inefficient because Pareto-improving resource allocation can be achieved by intertemporal redistribution of income. The paper is going to stress the importance of the dynamic inefficiency from the viewpoint of Japanese public debt sustainability later.

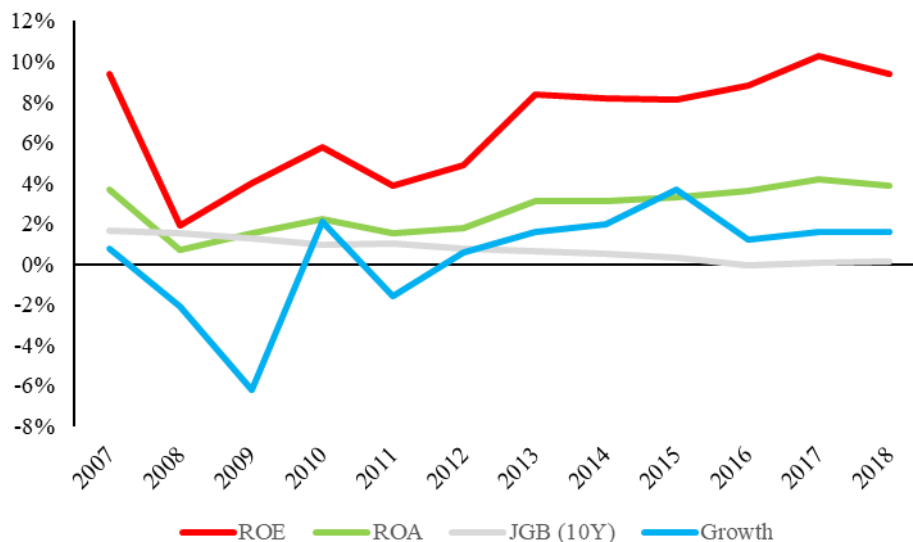
In dynamic inefficient economy where interest rate goes lower than growth, two respects are mathematically verified: 1) the golden rule introduced by Phelps (1961) reveals that the economic welfare increases even though the capital accumulation declines; and, 2) the overlapping generation model developed by Diamond (1965) results in the fact that issuing debt does, by decreasing capital accumulation, increase the welfare of both current and future generations. Tirole (1985) also insists possibility that some goods might exist as “bubble” which means that fundamentally valueless goods could take prices, and this “bubble” could improve resource allocation.¹⁰

Back to the debate in Japan in 2006, while the definition of growth rate is statistically clear, some confusion arose about the definition of the interest rate. In other words, in the CFP controversy, Mr. Takenaka had in mind the interest rate on public bonds, but on contrary, Professor Yoshikawa discussed the interest rate based on a neoclassical growth theory. In consideration of these factors, the paper firstly provides some theoretical basis of interest rates to ensure the accuracy of the discussion before beginning the survey on timeseries tests. First, we must take Ramsey (1928), which defines the interest rate as the sum of the growth rate and subjective discount rate. It is apparent that the interest rate is higher than the growth rate if the subjective discount rate is positive. Second, the interest rate based on neoclassical growth theory is that of the golden rule path in Solow (1956), which is equal to the growth rate. In other words, neoclassical interest rate is equivalent to that of Ramsey (1928) if the subjective discount rate is small enough. In both Ramsey (1928) and Solow (1956), the dynamic inefficient economy does not hold. Third, however in the scheme of the overlapping generation model introduced by Diamond (1965), the individual’s time horizon is limited and finite while the economy goes on forever going-concerned. And from the viewpoint of economic analysis, this overlapping generation model provides a theoretical basis for dynamic inefficient economy where the interest rate is lower than growth.

After considering the theoretical aspect of interest rate, empirical statistics will be discussed. Practically, in economy, we can observe some kinds of interest rates; 1) neutral interest rate which guides the economy to match saving and investment or demand and supply; 2) safe rates charged on government bonds (hereafter, JGB for Japanese case); 3) output return represented by ROA (return on assets) and/or ROE (return on equity); and, so on. We must select the most appropriate interest rate as “ r ” at Domar condition. Checking existing Japanese timeseries data, before COVID-19 pandemic, long-term JGB rate and time deposit rate are lower than ROA and ROE while growth rate is lower than ROA/ROE and higher than safe rates on JGB. Of course, these rates are partly subject to business cycle and some exceptional states are observed around the period of Lehman bankruptcy. Figure 4 shows ROE, ROA and JGB (10 year) and GDP Growth rates. Therefore, in Japan, following inequality seems to hold:

$$(EQ-4) \quad \text{Interest Rate} = r < \text{Growth Rate} = g < \text{ROA or ROE}$$

Figure 4: ROE, ROA and JGB (10 year) and GDP Growth rates



- source: 1) Ministry of Finance data for JGB (10 Year)¹¹
 2) Ministry of Economy, Trade and Industry data for ROE and ROA¹²
 3) Cabinet Office SNA data for GDP Growth¹³

This magnitude relationship observed in Japan also seems effective in the United States.¹⁴ Able et al. (1989), e.g., test the relationship between capital income and investment return instead of original relationship between interest rate and growth. According to its results, Japanese economy was regarded dynamic efficient at that time. In some economic literature, such as Blanchard (2019), Reis (2021), Barro (2023), Ball and Mankiw (2023), Kocherlakota (2023), and so on, public debt sustainability is analyzed under the economy that satisfy the magnitude relationship of (EQ-4).

This argument on relation between interest rate and growth looks important. Especially in Japan, many economists as well as statesmen focus strongly on the primary balance, i.e., the first term of the Domar condition and far small on the relation between interest rate and growth, i.e., the second term of righthand (EQ-3). Chapter 2 of Blanchard (2022), e.g., introduces five notions related to interest rates as follows:

- 1) neutral interest rate, defined as the safe real interest rate such that saving is equal to investment or such that aggregate demand is equal to potential output, assuming output is equal to potential output;
- 2) distinction between safe rates and risky rates such as the rate of return on stocks;
- 3) role of central banks to set the actual safe real interest rate as close as they can to the neutral interest rate;

- 4) importance of the inequality $(r - g) < 0$; ¹⁵ and,
- 5) nature and implications of the effective lower bound (ELB) of interest rate

3. Methodology for Public Debt Sustainability Test Using Timeseries Data

Finishing the discussion on interest rate and growth, this chapter deals with the methodologies for public debt sustainability using timeseries data. Looking back at the history of the economic theory, the fiscal management of the Reagan administration in the United States in the 1980s led to a large fiscal deficit, and the accumulation of public debt became a major opportunity to examine the sustainability of public debt. The first literature exploring this issue was Hamilton and Flavin (1986) based on the Domar condition refining Domar (1944). Hamilton and Flavin (1986) propose the budget constraint equation without taking the ratio of public debt over GDP as follows:

$$(EQ-5) \quad T - G - rD_{-1} + (D - D_{-1}) = B - rD_{-1} + (D - D_{-1}) = 0$$

where B primary balance¹⁶

(EQ-5) is regarded as difference equation and solved in a forward-looking manner, following (EQ-6) can be obtained:

$$(EQ-6) \quad D_t = \frac{B_{t+1}}{1+r_{t+1}} + \frac{D_{t+1}}{1+r_{t+1}} = \sum_{i=1}^{\infty} \frac{B_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} + \lim_{i \rightarrow \infty} \frac{D_{t+i}}{\prod_{k=1}^i (1+r_{t+k})}$$

Under the assumption that the future variables are subject to stochastic process, (EQ-6) is to be transformed into conditional expectation form, and following (EQ-7) can be obtained:

$$(EQ-7) \quad D_t = E_t \left[\sum_{i=1}^{\infty} \frac{B_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right] + \lim_{i \rightarrow \infty} E_t \left[\frac{D_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right]$$

(EQ-7) does not reject the possibility of exploding fiscal management including Ponzi game because it is obtained only transforming the government budget constraint equation. Therefore, one of following equations must be tested for public debt sustainability. This test provides a necessary and sufficient condition for the Ponzi game rejection in a dynamic model.

$$(EQ-8) \quad D_t - E_t \left[\sum_{i=1}^{\infty} \frac{B_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right] = 0 \quad \text{or}$$

$$\lim_{t \rightarrow \infty} E_t \left[\frac{D_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right] = 0$$

Intuitively, Hamilton and Flavin (1986) verify the primary balance of the government. After this pioneering literature, Trehan and Walsh (1988, 1991), Hakkio and Rush (1991), Haung (1991), Ahmed and Rogers (1995) and so on, introduce the cointegration method to test sustainability of public debt. According to Ahmed and Rogers (1995), at Domar condition of (EQ-1), G , T , and rD are tested in a cointegrated relationship. For this purpose, based on (EQ-7), shifting it by one period and taking the difference, following (EQ-8) can be obtained:

$$(EQ-9) \quad \Delta D_t = E_t \left[\sum_{i=1}^{\infty} \frac{B_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right] + \lim_{i \rightarrow \infty} E_t \left[\frac{D_{t+i}}{\prod_{k=1}^i (1+r_{t+k})} \right] - \lim_{i \rightarrow \infty} E_t \left[\frac{D_{t+i-1}}{\prod_{k=1}^i (1+r_{t+k-1})} \right]$$

Ahmed and Rogers (1995) precondition following three assumptions:

- 1) the second and the third terms of the righthand (EQ-9) are limit terms;
- 2) both G and T follow I(1) process;¹⁷ and
- 3) series of expected interest rate are constant during the test period.

Satisfying above three assumptions, two equations of (EQ-7) are equivalent to limit terms such as the second and the third terms of (EQ-9) being zero. Following equation of (EQ-10) is regressed for cointegrate regression of the Domar condition, i.e., (EQ-1) being tested whether G , T , and rD are cointegrated with cointegrate vector $[-1, 1, -1]$ or not:

$$(EQ-10) \quad T = a_0 + a_1 G + a_2 (rD) + u$$

where a parameter
 u error

Applying the methodology of Hamilton and Flavin (1986), some variations using timeseries data are introduced to economic literature: Wilcox (1989) tests using actual real interest rate as a discount factor instead of a constant expected rate in Hamilton and Flavin (1986); Blanchard et al. (1990) and Uctum and Wickens (1996), normalizing public debt by GDP, employ real interest rate deducted by real growth rate as a discount factor; and, Bohn (1995, 1998) adopt marginal substitution rate in intertemporal consumption as a discount factor, removing temporary and cyclical elements from government spending. Broda and Weinstein (2004) try to calculate required government revenue to maintain public debt sustainability in Japan after dividing government expenditure into two categories such as public pension payments and medical benefits for the elderly and the rest. The estimation

results reveal that approximate 35 percent of GDP is required for government revenue for the purpose of public debt sustainability and state that “the Japanese government’s target of trying to restore a primary balance of zero by 2012 is a particularly painful way of handling the transition for the current generation of workers.” (Broda and Weinstein 2004, p.41) All methodologies, referred so far here, are regarded as test for the primary balance. Among those surveyed in the paper, Bohn (1995, 1998) appear the most lenient methodology. Intuitively, the public debt is regarded sustainable when the primary balance deficits go shrinking. Doi and Nakazato (1998) test the central and local government bonds sustainability in Japan adopting the methodology of Bohn (1995) and result that Japanese fiscal system was sustainable at that moment.

Blanchard (2021) proposes stochastic debt sustainability analysis (SDSA) for testing debt sustainability under uncertainty of difference between interest rate and growth. The deficit gamble estimated in Ball et al. (1998) seems to correspond to this proposal. Under assumption of uncertain path of interest rate and growth, Ball et al. (1998) calculate gamble probability of public debt, adopting the primary balance ratio over GDP¹⁸ as data and Monte Karlo method as methodology. Using the same symbol so far, following another different Domar condition from (EQ-1) can be obtained adopting normalization of public debt by GDP:

$$(EQ-11) \quad \frac{D_t}{Y_t} = -\frac{B_{t-1}}{Y_{t-1}} + \frac{1+r_{t-1}}{1+g_{t-1}} \cdot \frac{D_{t-1}}{Y_{t-1}}$$

(EQ-11) can be solved in a forward-looking manner same as (EQ-6), and (EQ-12) is obtained presenting public debt ratio over GDP as follows with suffix t meaning period as usual:¹⁹

$$(EQ-12) \quad \frac{D_t}{Y_t} = -\frac{B_{t-1}}{Y_{t-1}} - \sum_{i=0}^{t-2} \left(\frac{B_i}{Y_i} \prod_{j=i+1}^{t-1} \frac{1+r_j}{1+g_j} \right) + \frac{D_0}{Y_0} \prod_{i=0}^{t-1} \frac{1+r_i}{1+g_i}$$

Concerning to righthand terms at (EQ-12), the first and the second terms strongly reflect the future primary balance ratio over GDP while the third term depends to a sizable extent on initial public debt ratio over GDP and the path of interest rate and growth. Ball et al. (1998) thus estimate the third term with Monte Carlo method while the first and the second terms promote the future improvement of the primary balance. With the definition of $X_t \equiv \frac{1+r_{t-1}}{1+g_{t-1}}$ and assumption that X_t follows AR(1) process, following (EQ-13) can be obtained:

$$(EQ-13) \quad X_t = \rho X_{t-1} + \sigma \varepsilon_t$$

where ρ autocorrelation coefficient

σ	standard deviation
ε	normal random number

Rewriting (EQ-12) with two definitions such as $X_t \equiv \frac{1+r_{t-1}}{1+g_{t-1}}$ and $d \equiv \frac{D_t}{Y_t}$, the public debt ratio over GDP, i.e., d is determined as follows:

$$(EQ-14) \quad d \equiv \frac{D_t}{Y_t} = -\frac{B_{t-1}}{Y_{t-1}} - \sum_{i=0}^{t-2} \left(\frac{B_i}{Y_i} \prod_{j=t+1}^{i-1} X_j \right) + \frac{D_0}{Y_0} \prod_{i=0}^{t-1} X_j$$

In (EQ-14), as mentioned before, the first and the second terms of the righthand can be removed under the assumption of $B = 0$. Practically, (EQ-13) and following (EQ-15) are to be estimated iteratively.

$$(EQ-15) \quad d \equiv \frac{D_t}{Y_t} \cong \frac{D_0}{Y_0} \prod_{i=0}^{t-1} X_j$$

Monte Carlo estimation can be applied for calculating the possibility that the initial public debt ratio over GDP, i.e., $\frac{D_0}{Y_0}$ exceeds a critical value at which the public debt would diverge or explode. Oguro (2009) employs this methodology for estimating the gamble probability in six developed countries and results that Japanese probability of gamble failure is the highest among those.

4. Exceptional Two Cases for Public Debt Sustainability

Yoshioka (2009) indicates two exceptional cases relating to public debt sustainability such as dynamic inefficiency and Ricardian equivalence: if one of these two holds, public debt goes sustainable unconditionally. First, dynamic inefficiency is focused on. The dynamic inefficiency is defined as the economic situation where interest rate²⁰ is lower than growth. From empirical viewpoint, in Japan, following two patterns are observed in 2000s as Figure 4 suggests:

- 1) Japanese economy is dynamic efficient when return on investment or equity is regarded as interest rate; and,
- 2) Japanese economy is dynamic inefficient when yield on government bonds is regarded as interest rate.

In natural, when the debt sustainability is focused on, the interest rate must be taken as the yield on government bonds. On the other hand, under the dynamic inefficient economy where interest rate is

lower than the rate of return on capital, capital accumulates in excess beyond the golden rule. And the Pareto optimum is achieved by reducing capital stock and increasing consumption according to standard macroeconomic textbook including Acemoglu (2009). In this sense, Japanese economy goes dynamic efficient since the return on capital assets (ROA) and/or the return on equity (ROE) exceed growth rate as Able et al. (1989) confirm. Practically in Japan, however, interest rate defined as yield on government bonds is lower than growth. For this Japanese case, this chapter checks public debt sustainability under some simple assumptions that three elements such as interest rate or yield on government bond (r), growth rate (g), and fiscal deficit rate over GDP (q) are constant across years.²¹

(EQ-16) Domar condition in another form

$$\begin{aligned} \Delta D &\equiv D_{+1} - D = G - T + rD && \text{then} \\ D_{+1} &= G - T + (1 + r)D && \text{taking ratio over GDP} \\ \frac{D_{+1}}{Y} &= \frac{G-T}{Y} + (1 + r) \frac{D}{Y} \end{aligned}$$

Substituting and rearranging definition of growth, i.e., $\frac{Y_{+1}}{Y} = 1 + g$, following (EQ-17) will be obtained:

$$(EQ-17) \quad (1 + g) \frac{D_{+1}}{Y_{+1}} = \frac{G-T}{Y} + (1 + r) \frac{D}{Y}$$

Expressing public debt ratio over GDP, i.e., $\frac{D}{Y}$ as x and assuming fiscal deficit ratio over GDP (q) are constant across years, following (EQ-17) will be obtained:

$$(EQ-18) \quad (1 + g)x_{+1} = q + (1 + r)x \quad \text{then}$$

$$x_{+1} = \frac{q}{1+g} + \frac{1+r}{1+g} x$$

where x public debt ratio over GDP, i.e., $\frac{D}{Y}$

q fiscal deficit ratio over GDP (constant across years)

In steady state, $x_{+1} = x = x_{-1} = \dots = x^*$ holds, following (EQ-18) will be obtained:

$$(EQ-18) \quad x^* = \frac{q}{g-r}$$

$$x_{+1} - x^* = \frac{1+r}{1+g} (x - x^*)$$

Thus, series $\{x_t - x^*\}$ is a geometric progression with the first term of $(x_0 - x^*)$ and term ratio of $\frac{1+r}{1+g}$, and following (EQ-19) will be obtained:

$$(EQ-19) \quad x_t - x^* = (x_0 - x^*) \left(\frac{1+r}{1+g}\right)^t$$

where x_0 initial value of x_t

For x_t to converge without diverging, the following necessary and sufficient condition (EQ-20) must be satisfied:

$$(EQ-20) \quad \frac{1+r}{1+g} < 1 \quad \text{i.e.,} \quad g > r$$

After confirming the first exceptional case for public debt sustainability, the second case deals with Ricardian equivalence. Bernheim (1987, p.264) insists that Ricardian equivalence denotes as follows:

The central Ricardian observation is that deficits (of the government)²² merely postpone taxes. A rational individual should be indifferent between paying \$1 in taxes today, and paying \$1 plus interest in taxes tomorrow. Since the timing of taxes does not affect an individual's lifetime budget constraint, it cannot alter his consumption decisions.

One of the simplest ways to understand Ricardian equivalence is to employ an overlapping generations model (OLG).²³ Without government, the budget constraints for both working and retired generations are expressed in following equations:

$$(EQ-21) \quad \text{working generation:} \quad Y = C_1 + S = C_1 + I$$

$$\text{retired generation:} \quad (1+r)I = C_2$$

where C_i consumption at each period
($i=1$ for working and $i=2$ for retired periods)

S savings

I investment or capital formation

Or two equations of (EQ-21) of the budget constraints will be expressed in following single equation:

$$(EQ-22) \quad Y = C_1 + \frac{C_2}{1+r}$$

Introducing government sector and assuming that entire government expenditure is financed by tax revenue, i.e., $G=T$, the budget constraint of (EQ-22) is transformed as follows:

$$(EQ-23) \quad Y - T = C_1 + \frac{C_2}{1+r}$$

On contrary, assuming that entire government expenditure is finance by bond issuance instead of tax revenue, and that interest rates or return on investment are identical with yield of government bond,²⁴ the budget constraint of (EQ-22) is transformed as follows:

$$(EQ-24) \quad \begin{array}{ll} \text{working generation:} & Y = C_1 + S = C_1 + (I + \Delta D) \\ \text{retired generation:} & (1 + r)(I + \Delta D) = C_2 \end{array}$$

Finally, assuming that the government taxes the same amount as the debt redemption to retired generation, budget constraint for the retired persons is expressed as follows:

$$(EQ-25) \quad \text{retired generation:} \quad (1 + r)(I + \Delta D) - (1 + r)\Delta D = C_2$$

Apparently, (EQ-25) is identical to the budget constraint for retired generation at (EQ-21). In short, government financing by tax revenue or bond issuance does not have any impact on consumption when Ricardian equivalence holds. Or the consumption is independent from types of government fiscal resources.

When Ricardian equivalence holds, the actions of households and other economic agents other than the government are independent of government funding and are not affected in any way. In other words, whether the government raises funds for government spending through taxes or through public bonds does not affect the behavior of economic agents other than the government, so public debt is unconditionally sustainable. This seems obvious. Dynamic efficiency is basically the relationship between the growth rate of the economy or population and the interest rate. When dynamic efficiency is not satisfied, i.e., if the interest rate is lower than the growth rate, intuitively thinking, government debt appears sustainable in the sense that public debt outstanding per capita or that as a percentage of GDP in the infinite future will not be so large as to diverge.

5. Why Japanese Government Do Not Go Bankrupt?

The government of Japan has been running a huge deficit²⁵ for long and public debt piles to an enormous amount as Figure 1 depicts. On contrary, few economists believe that Japan's government debt diverges or explode in near future. In fact, Japanese government seems to maintain full solvency and shows no sign of default of its bond. Before default, it is possible to take some preamble to lose solvency. Baldacci et al. (2011) and Gerling et al. (2017) insist that fiscal crisis before sovereign default indicates four distinctive criteria or combination of them: 1) credit events associated with

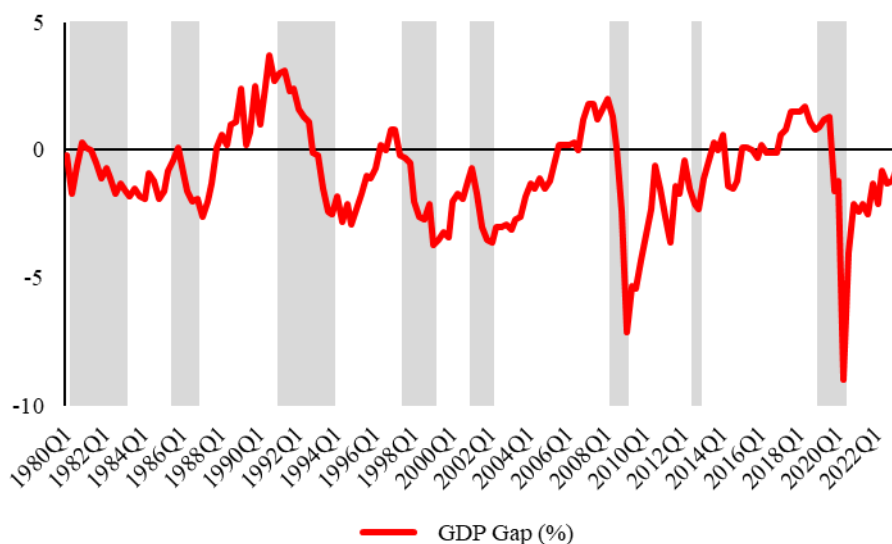
sovereign debt including outright defaults and restructuring; 2) recourse to large-scale IMF financial support; 3) implicit domestic public default, e.g., via high inflation rates; and, 4) loss of market confidence in the sovereign (Gerling et al. 2017, p.8). In Japan, these four criteria are not observed at all.

To strengthen government debt sustainability or to avoid default of government bonds, following four conditions are presented:

- 1) achieve a primary balance surplus or decrease a primary balance deficit;²⁶
- 2) satisfy Ricardian equivalence;
- 3) maintain difference between interest rate and growth that dynamic inefficiency runs; or
- 4) integrated government with the right to issue its own currency and a floating exchange rate system would not go bankrupt at all.

The first way to strengthen fiscal sustainability in Japan relates to the primary balance and seems so far satisfied after the Great Recession in 2008-09 except for COVID-19 period from 2020 to 2022 as Figure 2 shows. The primary balance deficits are decreasing in fact. But the fiscal consolidation is not the only economic policy target at all. It is not justified to continue strong austerity since the GDP gap in Japanese economy is still negative as Figure 5 reports.

Figure 5: GDP Gap in Japan



note: shadowed period indicates recession period.

source: Cabinet Office data²⁷ and the reference dates of business cycle²⁸

The second point to ensure the sustainability of Japan's debt is whether Ricardian equivalence holds or not. The former argument at the previous chapter reveals that Ricardian equivalence holds only on

a theoretical basis, and Poterba and Summers (1987) insist that Ricardian equivalence is to be regarded as a hypothesis. And apart from the viewpoint of debt sustainability, when Ricardian equivalence holds, the fiscal policy has no effect on economy. Auerbach et al. (2010), however, survey the impact of fiscal stimulation after the Great Recession and suggest negative view of invalidity of fiscal policy. Even limiting the topic on fiscal multiplier, Ramey (2011) estimates it to be closer to 1.2 while Christiano et al. (2011) investigate its size using DSGE model. Focusing on Japanese economy, Wu and Zhang (2010) apply the co-integrated and non-linear squares methods and address that Ricardian equivalence do not hold in Japan due to liquidity constraints. Walker (2002) results in mixed views suggesting that changes in expected permanent income due to government investment have effect on consumption but that the timing of taxes does not. It looks somehow unrealistic to assume that Ricardian equivalence holds in Japan.

The third topic relates to whether Japanese economy runs dynamic inefficient or not. Blanchard and Tashiro (2019) strongly insist that this point holds in Japan, e.g., “the profit rate has substantially decreased since the 1980s but remains substantially above Japan’s growth rate (Blanchard and Tashiro 2019, p.5).” Significantly, they insist that welfare costs depend on the balance of two relations: the relation between the safe rate and the growth rate on the one hand; and the relation between the average profit rate and the growth rate on the other. According to Blanchard (2019), based on the configuration of the interest rate, the profit rate, and the growth rate in Japan today and conditional on output staying at potential, higher debt has a small but positive welfare cost. Equivalently, lower debt has a small but positive welfare benefit. The paper already insists that the relation of (EQ-4) holds showing Figure 4 that depicts ROE, ROA and JGB (10 year) and GDP Growth rates. This third point therefore seems satisfied in Japan.

The fourth point is exactly what Modern Monetary Theory (MMT) suggests. MMT has been referred in economic literature by Wray (2015), Kelton (2020) and so on. Mochizuki (2019) addresses that MMT contains six important concepts: 1) Tax-Driven Monetary View; 2) Functional Finance; 3) Credit Monetary Theory and Monetary Circuit Theory; 4) Debt Hierarchy; 5) Stock-Flow Consistent Model; and 6) Job Guarantee Program. Among those, the second Functional Finance²⁹ relates strongly to the debt sustainability. Usually, MMT is regarded as a sort of macroeconomic school that asserts that sovereign nations,³⁰ including Japan, with spending, taxing, and borrowing in a fiat currency under full control, are not operationally constrained by revenues when government spends. Simply, MMT insists that such sovereign governments do not rely on taxes or borrowing for spending since they have the right to print as much money as they need but are constrained by stabilization of prices or control of inflation. Since their budgets are totally different from a regular household’s, their fiscal policies should not be restricted by fear of piling government debt and losing solvency. Moreover, Fullwiler (2007) addresses that implementation of MMT policies naturally results in a dynamic

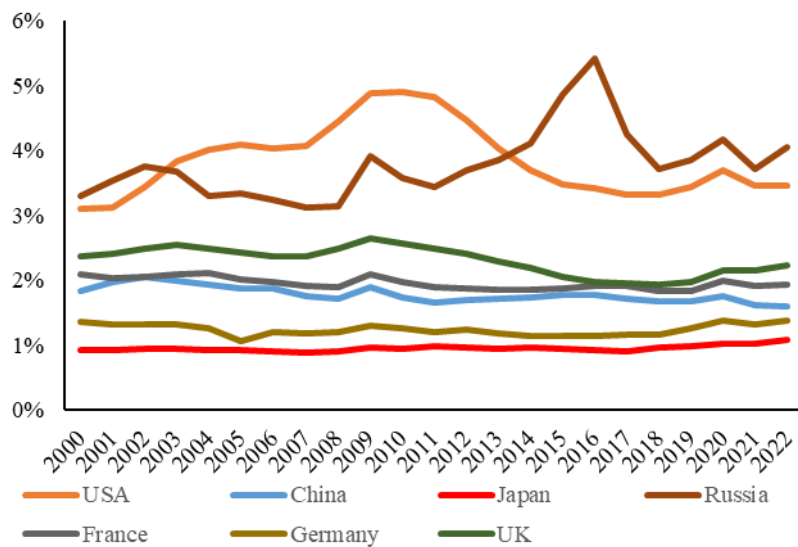
inefficient economy. However, regrettably, it is almost impossible to confirm whether this MMT is applicable to Japan or not.

Among above four factors that may play a significant role to maintain Japan's debt sustainability, Ricardian equivalence of the second point and the fourth respect, i.e., application of Modern Monetary Theory (MMT) is not practically acceptable. Not only from the viewpoint of elimination method, the first and the third suppositions such as the primary balance and the difference between interest rate and growth seem to support Japanese debt sustainability. According to CAO (2023), the primary balance of Japanese government will remain small negative but close to zero, and the public debt ratio over GDP will stabilize at just over 200-210 percent in the mid-2020s.³¹

6. Conclusion

The results of the paper suggest that Japanese government debt is positively sustainable. On the other hand, this conclusion does not mean that the government of Japan does not need to take any measures for the future debt sustainability. Because Japan must implement two important policies as the Kishida Cabinet presents. These two priorities for the future policy are as follows: 1) strengthening defense; and 2) countermeasures against the declining birthrate. First, in November 2023, Advisory Panel to Comprehensively Discuss Defense Capabilities as National Strength submitted the proposal to the Prime Minister Kishida to boost the defense budget drastically. This proposal is considered that the Government of Japan needs to double its military expenditure, which amounts to approximate one percent of GDP at present. Second, in April 2023, Countermeasure against Declining Birthrate on a Different Scale was revealed. The implementation of these two prioritized policies requires sizable government expenditure while the cabinet has not presented the additional revenue policy so far. Concerning to military expenditure, in February 2022, Russia invaded Ukraine and geopolitical risks are rising in East Asia as well while Japanese military expenditure has been the lowest among G-7 countries compared with the scale of GDP as Figure 6 reports. Some opinion leaders insist that these factors require more military preparation in Japan.

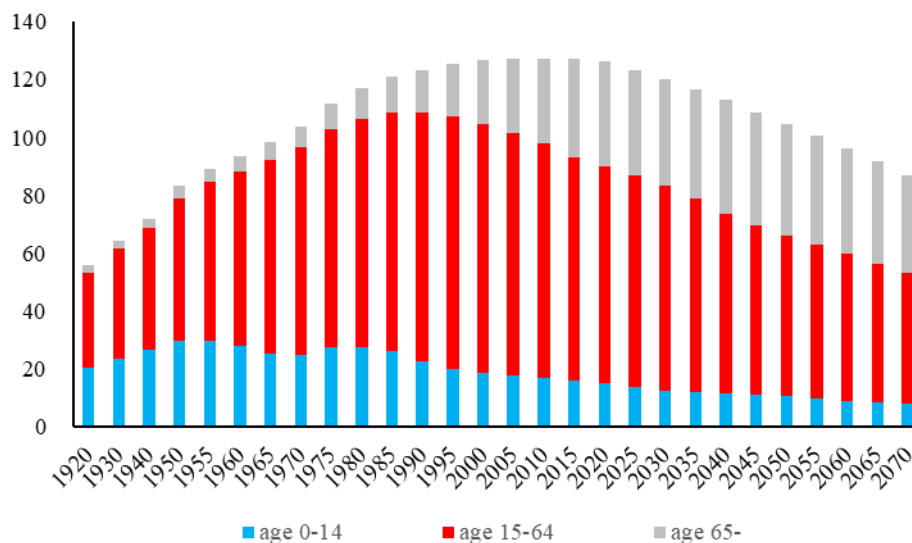
Figure 6: Military Expenditure as Percentage of GDP in Main Countries



source: SIPRI military expenditure database³²

Watching domestic economy, one of the most critical problems to be solved in Japan is obviously the declining birthrate and aging population. In fact, Japanese population will significantly decline in next fifty years according to Projection of Japanese Population estimated by National Institute of Population and Social Security Research. Its data do not indicate only population decline but also population aging as Figure 7 reports. A national consensus has been already established that some powerful countermeasures against the declining birth rate are absolutely required. The current cabinet embodies some countermeasures on a different scale. In nature, this kind of countermeasures must be backed up by affluent fiscal resources.

Figure 7: Population Projection and Structure by Age (million persons)



source: National Institute of Population and Social Security Research data³³

However, we must remember our experiences during 1990s and 2000s, i.e., so-called “lost decade” or “lost two decades.” Strong austerity brings downward pressure on national economy. Of course, it is out of scope of the paper to investigate reasons why Japanese economy ran depressed for long after the bubble burst in early 1990s. Exploring the reasons why the government debt ratio over GDP rose significantly after the bubble burst, however, CAO (2012) simply states that “a decline in revenues and an increase in social security costs contributed to the expansion of the primary fiscal deficit (CAO 2012, chapter 3 section2).” Thus it is worth to reveal that Eichengreen et al. (2021) regard the fiscal tightening as one of the most influencing causes of debt piling in Japan, addressing that brighter economy brought fiscal tightening and austerity caused slowdown of growth and decline of tax revenue (Eichengreen et al. 2021, chapter 11). In fact, the estimation of Ito (2003) reports that one percentage point decline of growth results in deterioration of 2.4 percentage points of government debt ratio over GDP compared with the previous year.

In short, the paper reveals following two points as the conclusion: 1) debt sustainability in Japan should depend more on the second term of Domar condition and less on the first term, i.e., difference between interest rate and growth is more important than the primary balance surplus to strengthen Japanese debt sustainability. This recommends the policy mix with combination of low interest rate and weak austerity; and 2) additionally, public debt sustainability with the consideration of the utility of public debt or fiscal deficit can be analyzed within the framework of mainstream economics without the help of heterodox economics such as Modern Monetary Theory (MMT) to a sizable extent.

With attention to the United States from Japan, in August 2023, Fitch Ratings downgraded the United States of America’s Long-Term Foreign-Currency Issuer Default Rating (IDR) to ‘AA+’ from ‘AAA.’³⁴ Its “Rating Watch Negative” was replaced to a “Stable Outlook” assigned while the Country Ceiling for the United States has been affirmed at “AAA.” Fitch Ratings addresses that this downgrade reflects three viewpoints: 1) expected fiscal deterioration over the next three years; 2) high and growing general government debt burden; and 3) the erosion of governance. On the other hand, Professor Krugman stated, “Fitch downgrades the U.S., a decision widely and correctly ridiculed.”³⁵ Currently, it is very difficult to determine which view is correct, but the financial markets so far have not overreacted. The market participants seem to regard US government bonds more sustainable than Fitch Ratings does.

At final, the paper cannot include three critical arguments: 1) fiscal theory of the price level (FTPL) that suggests price level and inflation is to be determined by fiscal policy developed by Leeper (1991), Sims (1994), Woodford (1995) and so on; 2) non-Keynesian effect that the tight fiscal management may possibly expand aggregate demand discussed at Giavazzi and Pagano (1990, 1995), Giavazzi et

al. (2000) and so on; and 3) verification of public debt sustainability adopting model simulation³⁶ such as employed in Braun and Joines (2015) and Hansen and Imrohroglu (2016). In particular, the first FTPL is rather important since MMT regards inflation as one of the most restrictive indicators to manage fiscal policy. On the other hand, the second non-Keynesian effect is not a theoretical interest but should be empirically verified. These issues including the third will be adopted in the future research.

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¹ In the paper, according to ordinary sense, “deficit” is referred on a flow basis while “debt” on a stock basis.

² Some economists, including Ocampo (2020), dispute the view of the functional fiscal policy because it is based partly on “the necessity for winning the war.” (Lerner 1943)

³

https://www.esri.cao.go.jp/jp/sna/data/data_list/kakuhou/files/2021/sankou/pdf/junkashi_20221223.pdf retrieved on August 4, 2023

⁴ For more detail on transversality condition, see Kamihigashi (2001, 2002, 2003).

⁵ Exactly, $G - T$ means the deficit of the primary balance or the negative primary balance.

⁶ Hereafter, simply “GDP” does not indicate real GDP or GDP at constant price but nominal GDP or GDP at current price in the paper.

⁷ Eichengreen et al. (2021) adopt an equation for debt analysis similar to Domar condition at appendix to chapter 7. They include the third term in the righthand named stock flow adjustment (SFA) term other than the first and the second terms of Domar condition. But this third term seems to contain rather small effect for Japanese debt. At chapter 11, they show Japanese debt factor decomposition. After the bubble burst in Japan, the government gross debt ratio over GDP rose from 64.5 percent in 1991 to 175.4 percent in 2007, which represents an increase of 111.9 percentage points in 16 years. According to Eichengreen et al. (2021), of this 111.9 percentage points, the primary balance contributed 66.6 percentage points, the difference between interest rate and growth 38.1 percentage points, and SFA only 7.2 percentage points. Hence, the paper does not include SFA term of Eichengreen et al. (2021) but employs Domar condition.

⁸ Some economists do not agree with Domar condition. For example, Yoshino and Miyamoto (2021) criticize that Domar condition is derived by focusing only on the supply of government bonds and not considering the demand for government bonds.

⁹ Exactly, ratio of the primary balance deficit over GDP, of course.

¹⁰ Tirole (1985) lists up some goods as possible “bubble,” such as fiat money, land, government bond, gold, and so on.

¹¹ https://www.mof.go.jp/jgbs/reference/interest_rate/data/jgbcm_all.csv retrieve on August 4, 2023

¹² https://www.meti.go.jp/shingikai/economy/sustainable_kigyo/pdf/001_05_00.pdf retrieve on August 4, 2023

¹³ <https://www5.cao.go.jp/j-j/wp/wp-je22/pdf/p040000.pdf> retrieve on August 4, 2023

¹⁴ Piketty (2014) strongly insists that $r > g$ holds, which causes inequality in income. But $r > g$ means ROA or $ROE > g$ in Piketty (2014).

¹⁵ On contrary, Blanchard (2022) seems to ignore the primary balance, i.e., $(G - T) < 0$.

¹⁶ Different from footnote 3, this primary balance is original.

¹⁷ Ahmed and Rogers (1995) propose random walk process with drift, e.g.

¹⁸ Ball et al. (1998) is also one of the variations derived from Hamilton and Flavin (1986) since testing public debt sustainability based on a primary balance.

¹⁹ Of course, suffix 0 means initial condition, e.g., D_0 and Y_0 in (EQ-12).

²⁰ Precisely, this interest rate must be deducted by capital depreciation.

²¹ Here, one of the simplest examples for debt sustainability based on relation between interest rate

and growth is reported. For more detailed discussion, see Darby (1984), Murata (1994) and so on.

²² The quoter adds remarks with parenthesized part., i.e., (of the government).

²³ Itagaki (2021) provides more detailed and generic explanation on Ricardian equivalence.

²⁴ Since savings are divided into two parts, i.e., capital formation and government bond, these two investments must be indifferent.

²⁵ When financing deficits, Japanese government has issued two types of government bonds: 1) construction bonds to finance government investment, such as infrastructure; and special deficit-financing bond to be stopgap measures. Ito and Hoshi (2020, p.197) address that “there is no difference between two types of bonds in terms of government obligation.”

²⁶ See Bohn (1995, 1998).

²⁷ <https://www5.cao.go.jp/keizai3/getsurei/2312gap.xlsx> retrieved on August 4, 2023

²⁸ <https://www.esri.cao.go.jp/en/stat/di/di2e.html> retrieved on August 4, 2023

²⁹ Drumetz and Pfister (2021a, 2021b) address that the functional finance of Modern Monetary Theory is based on Lerner (1943).

³⁰ Here, sovereign nations are presumed as: 1) integrated government including both executive branch of the government in a narrow meaning and the central bank with the right to issue its national currency; and, 2) adoption of a floating exchange system.

³¹ Here, it is recommended to focus more on the flow data of the primary balance than on the stock data of debt. Yoshioka (2010) addresses that the former is determined exogenously by the government while the latter is endogenously calculated based on the former.

³² <https://www.sipri.org/databases/milex> retrieved on August 4, 2023

³³ https://www.ipss.go.jp/pp-zenkoku/j/zenkoku2023/db_zenkoku2023/s_tables/1-1.xlsx retrieved on August 4, 2023

³⁴ <https://www.fitchratings.com/research/sovereigns/fitch-downgrades-united-states-long-term-ratings-to-aa-from-aaa-outlook-stable-01-08-2023> retrieved on August 4, 2023

³⁵ <https://twitter.com/paulkrugman/status/168649745275501056> retrieved on August 4, 2023

³⁶ Braun and Joines (2015) employ an overlapping generations model while Hansen and Imrohroglu (2016) utilize a neoclassical growth model. Both test Japanese debt sustainability and suggest pessimistic results.